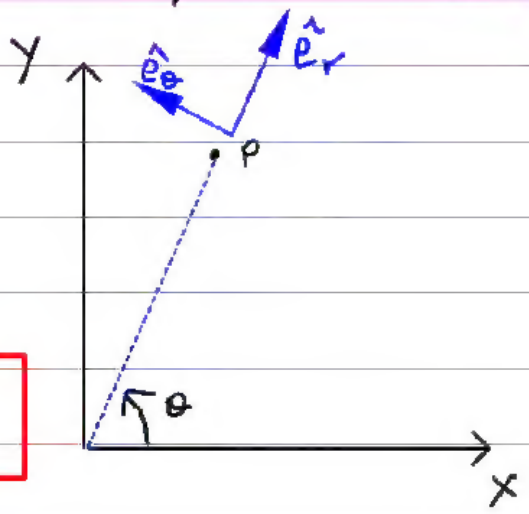
Transferring vectors from one coordinate system to another

Eg.

point P is at {3,8} meters

Its velocity is $4\hat{e}_r + 3\hat{e}_\theta$ m/s.



Find its velocity in x-y coordinates

1/ Find the direction of $\hat{e}_r$, $\hat{e}_\theta$:

$\theta = \tan^{-1}\left(\frac{8}{3}\right) = 69.4^\circ$

2/ → angle between x-axis and $\hat{e}_r$ is 69.4°

angle between y-axis and $\hat{e}_r$ is $69.4^\circ - 90^\circ = -20.6^\circ$

angle between x-axis and $\hat{e}_\theta$ is $90^\circ + 69.4^\circ = 159.4^\circ$

angle between y-axis and $\hat{e}_\theta$ is $159.4^\circ - 90^\circ = 69.4^\circ$

3/ $4\text{m/s}\ \hat{e}_r = 4\text{m/s}\left(\hat{i}\cos 69.4^\circ + \hat{j}\cos(-20.6)\right) = 1.41\,\hat{i} + 3.74\,\hat{j}$

$3\text{ m/s}\ \hat{e}_\theta = 3\text{m/s}\left(\hat{i}\cos 159.4 + \hat{j}\cos(69.4)\right) = -2.81\,\hat{i} + 1.06\,\hat{j}$

4/ Totals:

$\vec{v} = -1.4\,\hat{i} + 4.8\,\hat{j}$ m/s

5/ check: $|\vec{v}|$ in r-θ system $= \sqrt{3^2+4^2} = 5$ m/s.

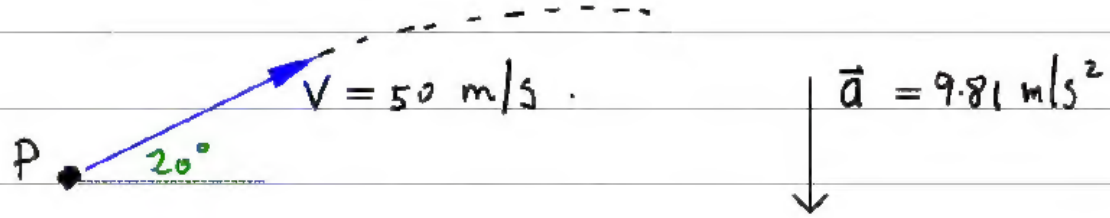
$|\vec{v}|$ in x-y stem $= \sqrt{(-1.41)^2 + (4.8)^2} = 5.00$ m/s

Summary

To move from one coordinate system to another:

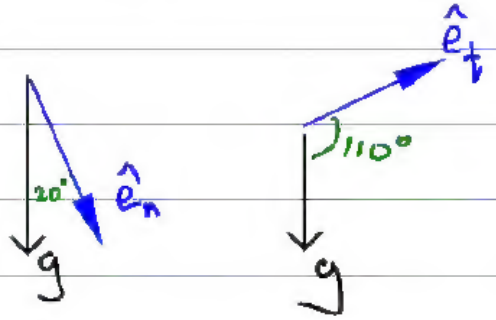
PROJECT the known vector onto the desired axes.

e.g. what is a_n, a_t?



1/ Find the normal and tangential directions

2/ Find angles between known acceleration and desired axes.



3/ Project the given acceleration onto the +ve n-t axes

$$a_n = 9.81\cos(20^\circ) = 9.22\ m/s^2$$
$$a_t = 9.81\cos(110^\circ) = -3.36\ m/s^2$$